Generalized way to find individual digits during number conversion

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Theorem

Given that a number in radix 10, n, is to be converted into radix b, the kth digit of $(n)_b$ is found by the function below:

$$\delta(n, b, k) = \left\lfloor \frac{n}{b^k} \right\rfloor - b \left\lfloor \frac{n}{b^{k+1}} \right\rfloor, \qquad n \geq b^k, \quad k \in \mathbb{N}, \quad n, b \in \mathbb{R} \tag{1}$$

Proof

When converting from radix 10 to another base, a division using remainder can be performed. For example, when 7 should be converted into radix 2, the following algorithm can be used:

1. Divide 7 by 2 to get 3 with a remainder of 1.
2. Divide 3 (from (1)) by 2 to get 1 with a remainder of 1.
3. Collect the last answer (i.e. in (2)) and all the remainders backwards. That is 'last answer', remainder in (2)', 'remainder in (1).
4. The result should be $(111)_2$.

The algorithm above can be generalised as follows:

1. Find the remainder when number n is divided by base b, e.g. $n \bmod b$.
2. Use the quotient in (1) to find the new remainder when the quotient is divided by the base, e.g. $\lfloor n/b \rfloor \bmod b$.
3. Repeat step (2) to get the second digit, e.g. $\left\lfloor \frac{\lfloor n/b \rfloor}{b} \right\rfloor \bmod b = \lfloor n/b^2 \rfloor \bmod b = \lfloor n/b^2 \rfloor - b\lfloor (n/b^2)/b \rfloor = \lfloor n/b^2 \rfloor - b\lfloor n/b^3 \rfloor$
4. The step (3) should be repeated as long as $b^k \leq n$.

$$\therefore f(n, b, k) = \left\lfloor \frac{n}{b^k} \right\rfloor - b \left\lfloor \frac{n}{b^{k+1}} \right\rfloor, \qquad n \geq b^k, \quad k \in \mathbb{N}, \quad n, b \in \mathbb{R}$$